Final Exam — Complex Analysis (WBPH059-05)

Monday 16 June 2025, 11.45h–13.45h

University of Groningen

Instructions

1. The use of calculators, books, or notes is not allowed.

2. Provide clear arguments and computations for all your answers: only answering "yes", "no", or "42" is not sufficient.

3. The total score for all questions equals 90 points. If p is the number of marks then the exam grade is $G = 1 + p/10$.

Problem 1 (18 points)

For numbers $a, b \in \mathbb{R}$ consider the following function:

$$f(z) = (a + bi)\,\mathrm{Re}(e^{iz}) - \mathrm{Im}(e^{iz}).$$

Use the Cauchy-Riemann equations to determine all values of $a, b \in \mathbb{R}$ such that f is differentiable for all z.

Problem 2 (18 points)

Let C be the circle $|z| = 2$ traversed once in the positive sense. Use any method of your choice to compute the following integral:

$$\int_C \frac{\cos(z)}{z^2(z-4)}\,dz.$$

Problem 3 (9 + 9 = 18 points)

Compute the Laurent series for $f(z) = \dfrac{z^2 - 2z + 3}{z - 2}$ in the following domains:

(a) $|z - 1| > 1$;

(b) $|z - 2| > 0$.

Problem 4 (12 points)

Classify the singularity of $f(z) = \dfrac{1}{1 - \cos(z)}$ at $z = 0$.

Problem 5 (24 points)

Use the method of residues to compute the following integral:

$$\mathrm{p.\,v.} \int_{-\infty}^{\infty} \frac{x^2 - 1}{(x^2 + 4)(x^2 + 9)}\,dx.$$

End of test (90 points)